Grid Method Classification of Islamic Geometric Patterns

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ABSTRACT

This paper proposes a rational classification of Islamic Geometric Patterns (IGP) based on the Minimum Number of Grids (MNG) and Lowest Geometric Shape (LGS) used in the construction of the symmetric elements. The existing classification of repeating patterns by their symmetric groups is in many cases not appropriate or prudent [Joy97]. The symmetry group theories do not relate to the way of thinking of the artisans involved, and completely has ignored the attributes of the unit pattern and has focused exclusively on arrangement formats. The paper considers the current symmetric group theories only as arrangement patterns and not as classifications of IGP since they have a "global approach" and have failed to explore the possibilities in the construction elements of IGP. The Star, a central Rosette, which is the most important element of IGP, forms the core of our study. The paper proposes new nomenclature to be used in the description of the unit pattern based on the MNG and LGS used in the construction of a Star/Rosette pattern that can be used to achieve the final design. We describe and demonstrate procedures for constructing Star/Rosette unit patterns based on our proposed classification in a grid formation dictated by the final design of the unit pattern.

Keywords :- IGP, MNG, LGS, Grid, Classification, Group Theory, Star/Rosette.

1. INTRODUCTION

Islamic artisans began to adorn the surfaces of palaces, mosques and minarets with IGPs more than thousand years ago [Sai76]. The geometric designs consistently filled the surface planes with star-shaped like regions that resulted in very highly visual symmetric patterns, which would henceforth be referred to as "Islamic Geometric Patterns". These geometric patterns have often presented a long-standing historic awe to group theorists who have endeavoured to present a prudent classification of these structures. Many attempts that have been made to classify the Star/Rosette patterns have resulted in a wide variety of construction groups and classifications. Grunbaum and Shephard tried to decompose these geometric patterns by their symmetry groups after obtaining the base region, which they had used to arrive at the properties of the original pattern [Gru92]. European group theorists like Dewdney have proposed a classification based on reflecting lines off to periodically placed circles [Dew93]. Lee has presented simple constructions for the common features of IGPs but has failed to present a benchmark classification theorem [Lee95]. Also one important aspect of IGPs that has failed to attract any kind of classification is the naïve extension of lines into interstitial regions. We have had a good look into the existing complexity to the inferred geometry in order to describe the relation of the extension region with the unit pattern. S.J. Abbas and A. Salman in their landmark thesis on *A Symmetries of Islamic Geometrical Patterns* were of a firm opinion that no worthwhile classification of IGPs has been taken up to this day with specific focus on their construction [Abb95]. This paper presents an argument that popular and existing symmetry groups like the "7-frieze groups" and the "17-wallpaper groups" are purely base models. A more finer and refined classification based on the study of the construction of the unit pattern is required with specific focus on the gridding system of the unit pattern. The accomplishment of the unit pattern by the MNG, LGS and the infinite number of possibilities this classification presents by way of permutations and combinations of the grids are huge.

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2. SYMMETRY

Symmetry means a balance, a repetition of parts or simple uniformity of form. Symmetry simply means pattern. But the range of symmetry is far more than simply appealing architecture and pretty patterns. However, mathematically symmetry can be simply defined in terms of invariance of properties of sets under transformation [Abb92]. Group theory shows that in a single dimension symmetric period pattern can be analysed into seven different types and provides the information needed to identify a particular symmetry type [Abb95]. Also, in a two dimension symmetric period pattern, seventeen different types of patterns can be generated and identified. The Single dimensional symmetric pattern is referred to as "7-frieze groups" and the double dimensional symmetric pattern is referred as the "17-wallpaper groups". This paper presents a visual input into the powerful notions of pattern and symmetry in IGPs by analysing the construction of the individual element of the symmetric patterns. The Following sections describe existing and conventional symmetric group theories to endorse our view that they are merely arrangement patterns rather than classification theories of geometric patterns let alone IGPs.

3. THE 7-FRIEZE GROUPS

Isometric groups that keep a given straight line invariant including translations along the line are called frieze groups. Isometric may be defined as a linear transformation of the plane or space that preserve distances between points. To illustrate the 7-frieze groups argument (see figure 1) we present the attributes of each frieze group by Andrew Glassner [Gla99] who has done some very inspiring research into many related topics like frieze groups, moiré patterns, mirror reflection and a periodic tiling. He has demonstrated the value of creating physical models that enabled us to stretch our visualization skills and also our perception of the subject matter. It should be very important to note that in a very definitive sense that the frieze group theories misdirect classification of IGPs. Mathematicians find it very convenient and useful to interpret regularity of a pattern in terms of its group of symmetry. In this way the results of algebra and other mathematical disciplines can be applied to the study of such patterns. However, it could be argued that this is not the concept of regularity that artisans had in mind as they were creating their art. In fact, until a century or so ago, even to mathematician's regularity of mathematical objects had a completely different meaning. The difference between the two approaches is to a large degree the contrast of the global and local points of view. Mathematicians used to define regularity of objects such as Platonic polyhedral by requirements of congruent faces, equal angles, and other local properties, now it is customary to define regularity by the transitivity of the symmetry group on the set of flags. In the same way, it seems likely that the artisans meant to create geometric patterns in which part is related to its immediate neighbours in some specific way and not by attempting to obtain global symmetries of the infinitely extended design.

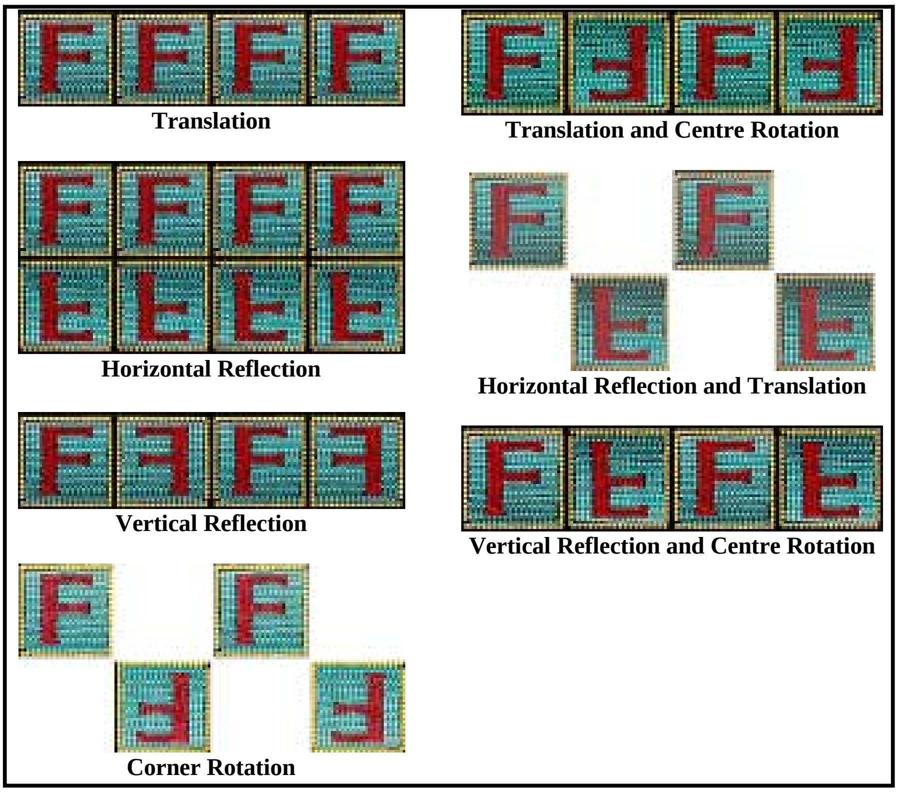

Figure 1. The 7-frieze groups by Andrew Glassner [Gal99]

4. THE 17-WALLPAPER GROUPS

It has been established that 17 distinct groups of two-dimensional patterns that are periodic in two independent directions exist. These 17 patterns are also popularly referred to as the 17-wallpaper groups. However, the essential nomenclature that has been assigned to the 17-wallpaper groups may be given by Xah Lee [Lee98] (see figure 2). David E. Joyce [Joy97] from Clark University in his Internet site on the 17 plane symmetry groups regarded symmetry groups as classification of planar patterns. He wrote as "the various planar patterns can be classified by the transformation groups that leave them invariant. A mathematical analysis of these groups shows that there are exactly different plane symmetry groups". Now, it is very clear from the above illustrations that the frieze and the wallpaper group's theories present arrangement benchmarks, which enable us to determine the type of format pattern arrangement rather than classify the unit pattern. Also we can conclude that a viable and arguable approach has not been taken up with a holistic view to classify IGPs with specific focus on their construction.

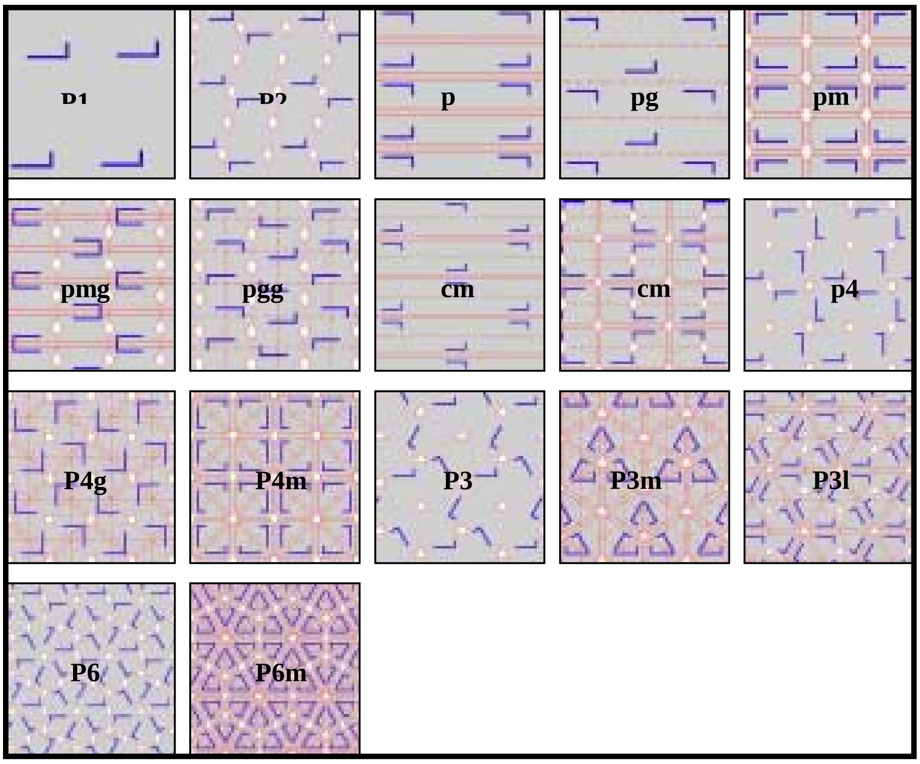

Figure 2. The 17-wallpaper groups by Xah Lee [Lee 98]

5. GRID METHOD CLASSIFICATION

The objective of this paper is to propose a new classification based on the construction of the unit pattern of IGP as it has been proved that existing group theories do not serve this purpose.

Usually any given IGP is named on the basis of its given geometric shape. For example, illustrations by Issam El-Said [Sai93] the pattern could be classified as a hexagonal pattern simply because it contains hexagonal star or could be classified as an octagonal pattern because it contains octagonal star, etc. (see figure 3). But this could be misleading, because the Star/Rosette the most popular element of most of the IGP may be accomplished by a combination of several geometric shapes like circles, triangles, squares, quadrilaterals and hexagons etc., where a Star/Rosette unit patterns could be normalised and classified according to its basic design. Therefore, instead of looking at these images as a hexagonal or octagonal like unit patterns, we would like to classify these images based on the construction and normalization of the gridding of the Stars/Rosette. And look at the important attributes and properties of a given Star/Rosette. In our method, any given Star/Rosette can be deciphered or de-constructed by normalising it. This normalisation process would be achieved by identifying individual grids that make up the Star/Rosette. Once grid elements are separated the basic geometric shape that can be used to achieve the n-gon Star/Rosette would be identified. The process of dissection of a star according to us could be taken up in the following stages:

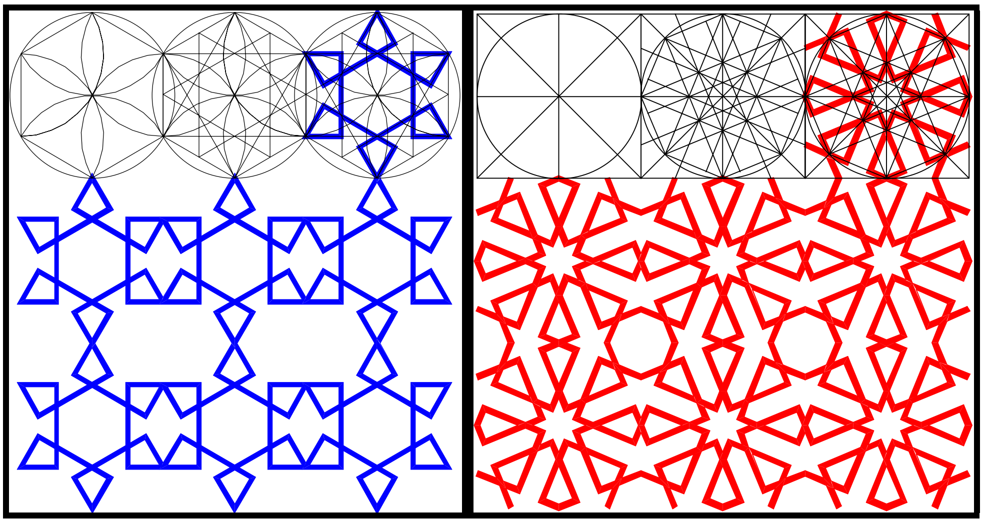

Figure 3. Hexagonal and octagonal patterns by Issam El-Said in "*Islamic Art and Architecture, The System of Geometric Design*" *[Sai93]*.

5.1 The planar surface stage

This is the basic unit circle or the planar surface on which the grids would be placed to achieve an n-gon unit star. Its radius strictly restricts the placing of the grids within the parameters of its size. Here we have a very strong difference of opinion with W. K. Chorbachi [Cho89] who in his landmark effort *"Tower of Babel: Beyond Symmetry in Islamic Design"* has said as following:

> "…. in *Geometric Concepts in Islamic Art* by I. El-Said. In his introduction to it, Titus Burckhardt states that all the geometric patterns are derived by the same method of deriving all the vital proportions of a building (or a pattern) from the harmonious division of a circle…. In some cases however, the authors neglected to draw in the circle, ironically revealing how unfundamental its existence is to the alleged "unique way" or "only way" of deriving all patterns (see figure 4.2) [Sai76].

We were able to discover the same image (see figure 5) with the base circle marked in a clear manner in El-Said's *Islamic Art and Architecture, The System of Geometric Design* [Sai93] which shows that El-Said did not ignore to draw the unit pattern in the circle. In this context, we can say that (figure 4) has been viewed from a different and convenient dimension so as to facilitate a suitable conclusion to justify the theorem that was being presented by W.K. Chorbachi. The circle in (figure 5) which is a replica of (figure 4) indeed forms the base planar surface for the design of the unit pattern, as is what this paper is arguing.

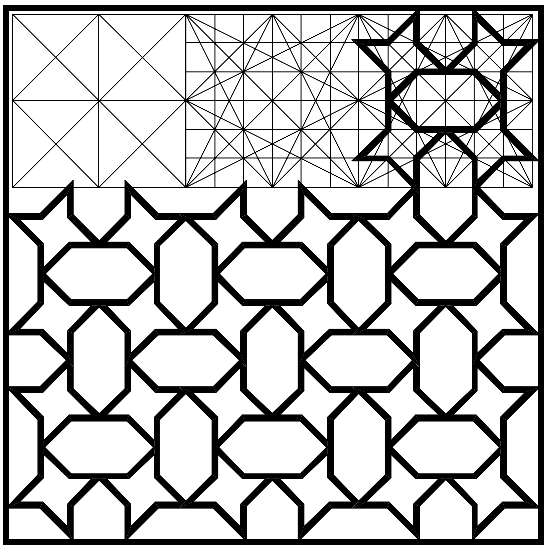

Figure 4. This image is Fig. 4.2 From W. K. Chorbachi, *Tower of Babel, Beyond Symmetry in Islamic Design [Cho89]*, showing that the circle does not appear in deriving this pattern. From I. El-Said, *Geometric Concepts in Islamic [Sai76]*.

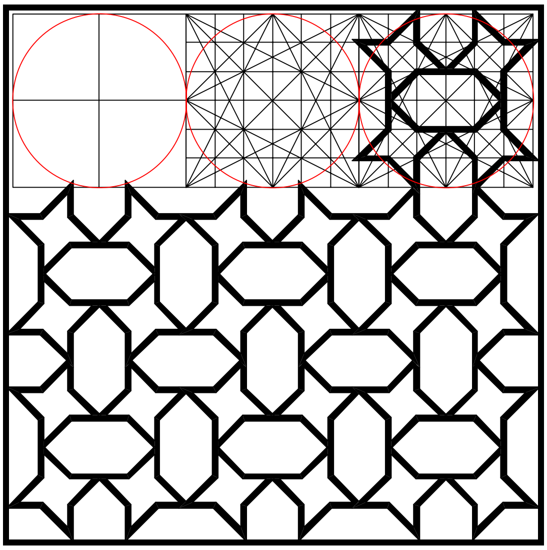

Figure 5. The circle does appear in the unit pattern. From I. El-Said, *Islamic Art and Architecture, The System of Geometric Design [Sai93].*

5.2 The divisional stage

Here we divide the circle (360 degrees) by x number of points to arrive at the intended design of the Star/Rosette.

5.3 The gridding stage

The gridding stage would initiate the gridding process. This stage is the most important stage in the chronological stages that have been stated by this paper. It has been observed that design formats of Stars/Rosettes found in IGPs are varied and very different to each other. Since Islam itself is spread across so many continents and each country has contributed its own artistic heritage to Islamic Art. In this background, to properly decipher a star pattern one must make a very properly guided endeavour in order to know the type of Star/Rosette. We know the complexity of this task because the very nature of Islamic art is very intricate and any intricate art is difficult to normalise. The core objective of this stage is to describe and classify the Star/Rosette with reference to the Minimum Number of Grids (MNG) and the Lowest Geometric Shape (LSG) used in achieving the design of the IGPs.

I have chosen an unusual and complicated pattern to demonstrate our method of classification (see figure 6). The pattern is an extract from; The Mathematical Gazette *"Some Difficult Saracenic Designs, A Pattern Containing Fifteen Rayed Stars"* by E. Hanbury Hankin [Han36]. In any given unit pattern requires to be classified according to our method, we start by looking for the different types of Stars/Rosettes in the given unit pattern. I have taken the liberty of colouring the unit pattern given by Hankin to show the different types of Star/rosette in the given unit pattern. The given unit pattern is unusual because it consists of two types of stars which are similar in type but different in design; one is twelve rayed star and small in size and the other is fifteen rayed star and larger. What makes the pattern extra ordinary is that the two different sizes of stars beautifully connected to each other with sets of meshes of lines between them. The size of star doesn't effect our classification rather the method of design of each Star/Rosette and its gridding attributes. Therefore we conclude that the first attribute of this given unit pattern is that it consists of multiple Stars/Rosettes.If we start to classify the unit pattern by Hankin, according to the standards or norms of the conventional frieze and wallpaper group theories we would have to take the enclosed area as the primary unit pattern. By doing so we are by-passing the finite elements or finite properties of the image. The following sections elaborate in very fine detail the process of normalising the construction of IGPs based on the elements of grids (MNG) and the properties of grids themselves (LGS).

5.3.1 Minimum number of grids (MNG)

This section would initiate the first part of our naming convention which is aimed at identifying the minimum number of set of grids mounted above each other to achieve an n-gon whose vertices bisect its edges. This section takes the end design as its core objective. An infinite loop process goes on identifying intersections and sets the correct relationship with vertices to achieve bisections of the rosette.

5.3.2 Lowest geometric shape (LGS)

This stage initiates the second part of our naming convention which is aimed at identifying the lowest possible geometric shape that is used to construct the Star/Rosette within the given unit pattern. The following illustrations (see figures 7 and 8) describe the normalisation series of the pattern (see figure 6) to achieve the classification of the twelve and the fifteen-rayed Star/Rosette respectively.

The twelve-rayed Star/Rosette uses 3 Minimum Number of Grids (MNG) and a Quadrilateral as the Lowest Geometric Shape (LGS) where as the fifteen-rayed Star/Rosette uses 3 Minimum Number of Grids and a Pentagon as the Lowest Geometric Shape. Therefore we classify this pattern (see figure 6) as:

Grid 3 Quadrilateral/Pentagonal Class.

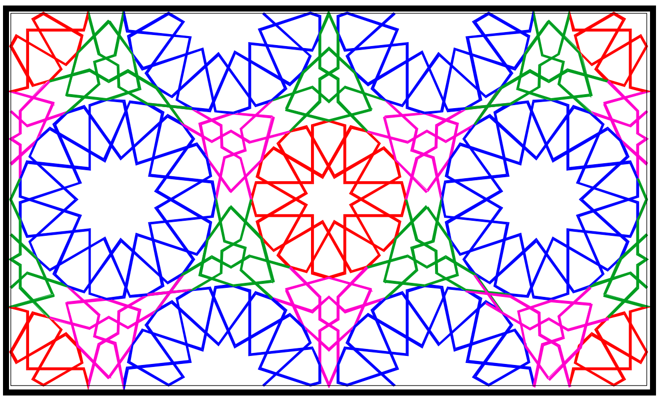

Figure 6: Pattern containing twelve (red) and fifteen (blue) rayed Star/Rosette by E. Hanbury, *Mathematical Gazette "Some Difficult Saracenic Designs" [Han36].*

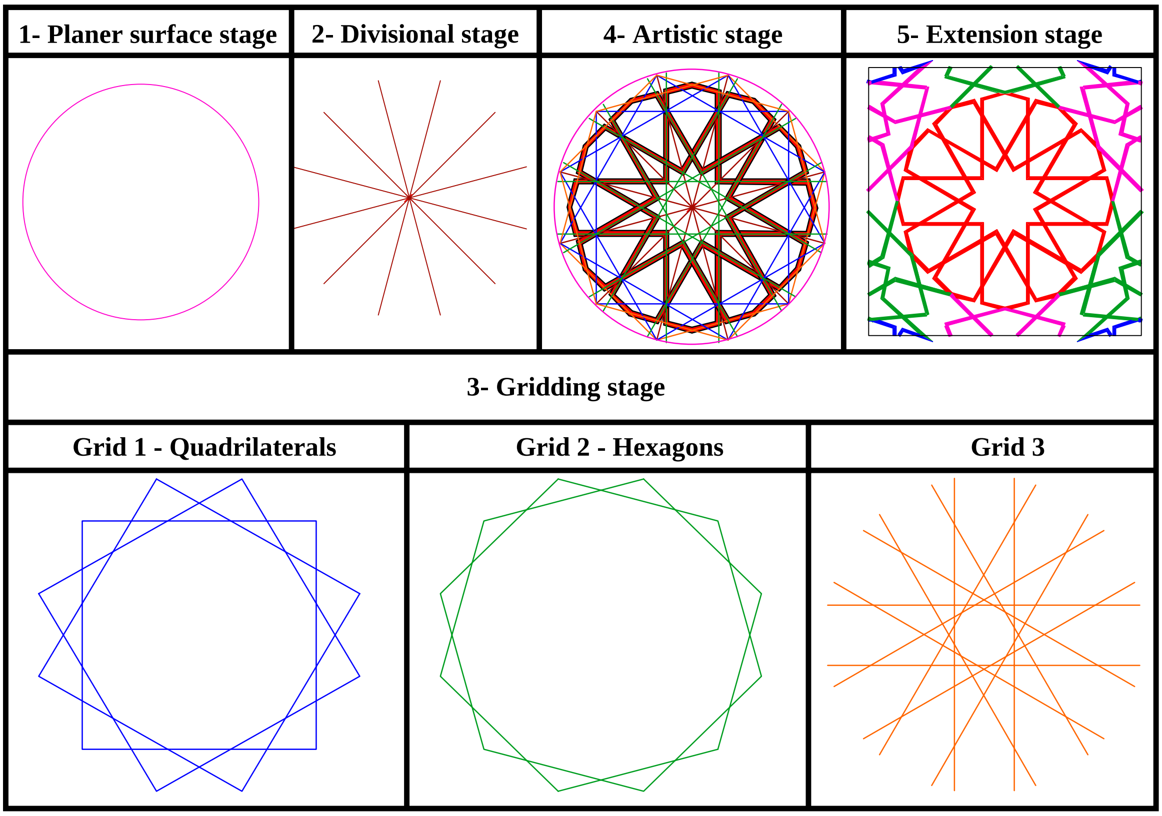

Figure 7. The 12-rayed star is classified as (Grid 3 Quadrilateral Class) because it uses minimum of 3 sets of grids and the lowest geometry is quadrilateral.

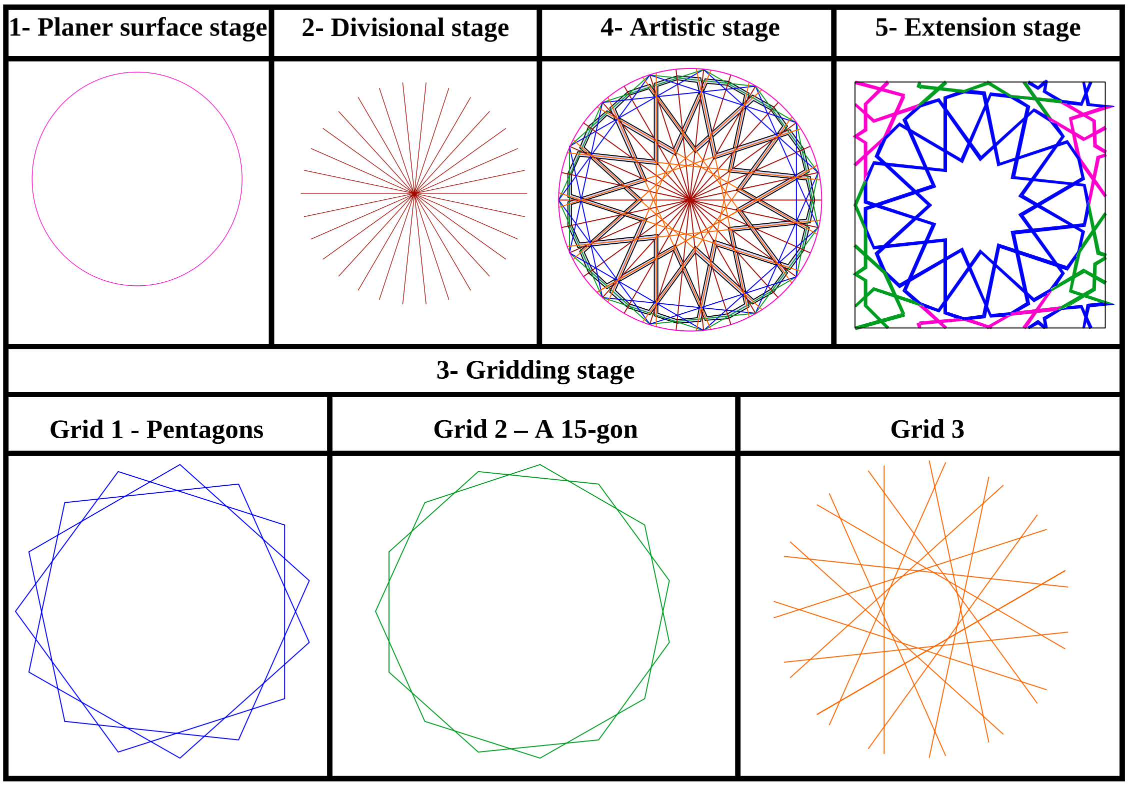

Figure 8. The 15-rayed star is classified as (Grid 3 Pentagonal Class) because it uses minimum of 3 sets of grids and the lowest geometry is a pentagon.

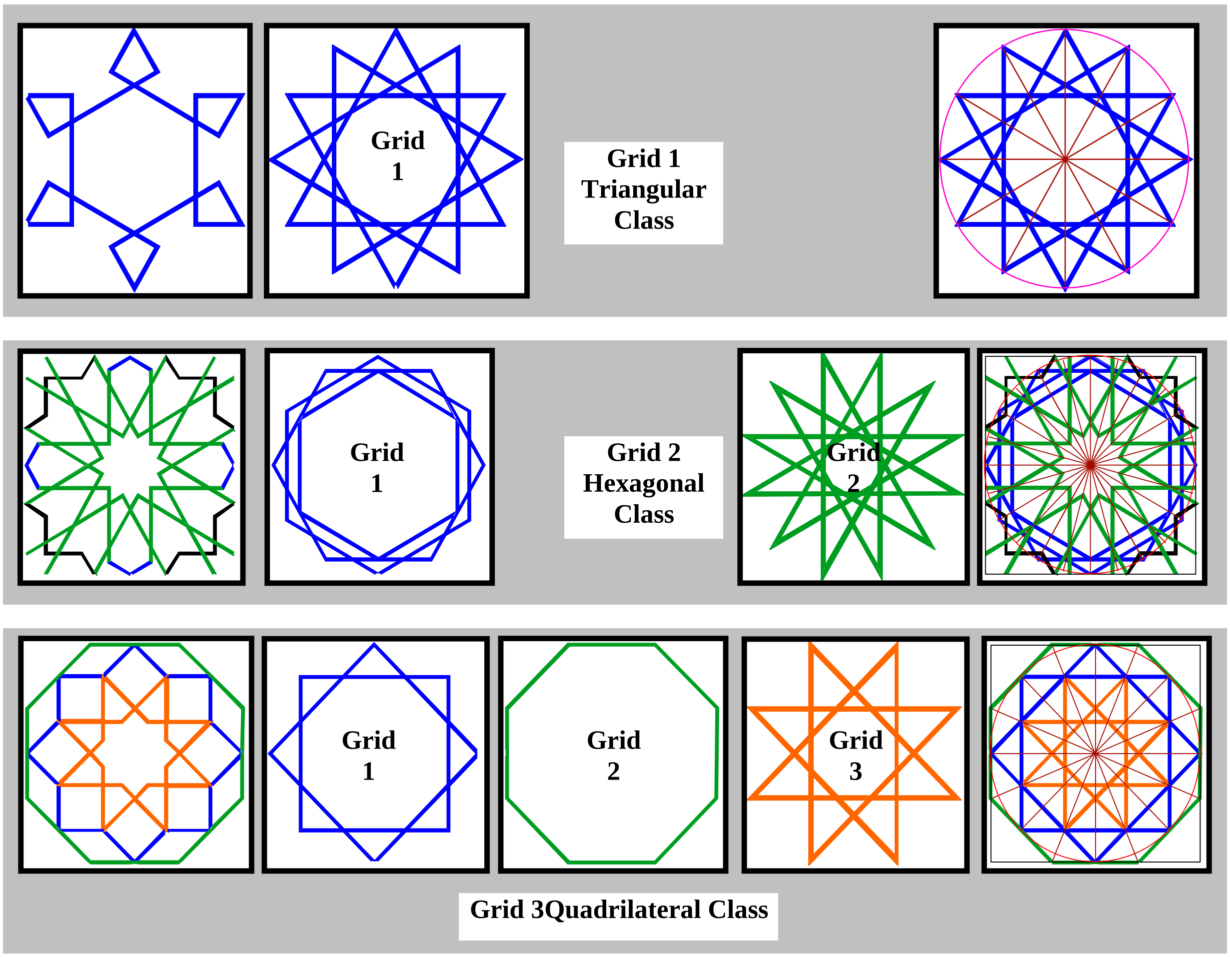

Figure 9. Showing the classification of some Islamic Geometric Star/Rosette.

Following these sections are series of images that would illustrate the Normalization process in a very logical and visual manner that would put forth our point in a presentable manner (Figure 9).

5.4 The artistic stage

This is the fourth stage; here once the gridding is achieved we may design the intended Star/Rosette by giving the necessary artistic attributes to the grid by way of presenting weights to the internal lines of the grid. This stage may also include colouring and filling the sections of the Star/Rosette.

5.5 The extension stage

The fifth stage is a Notional or Phantom stage because this stage might exist or might not exist. In this stage the natural extensions would evolve to accomplish the seamless mesh in the external zone within the notional boundary (usually a square or a rectangle) and beyond (see figure 7 and figure 8).

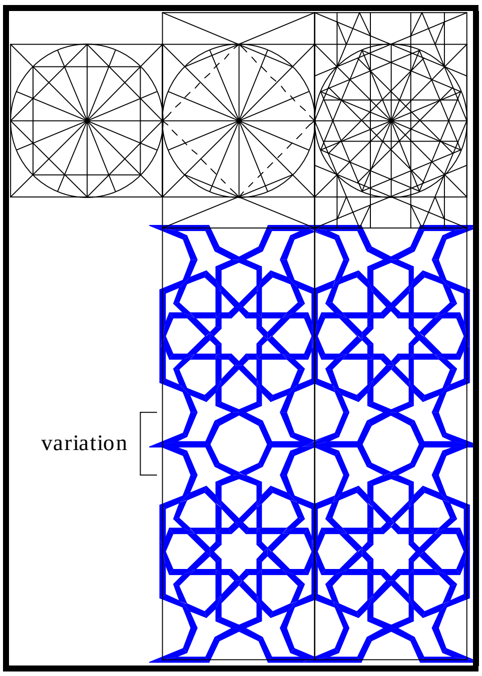

Figure 10. This image is Fig. 4.3 from W. K. Chorbachi *[Cho89]*, showing that the scheme of the circle does not fit the long rectangular unit, a variation zone is marked. From I. El-Said *[Sai76]*.

El-Said [Sai93]has regarded the square as a definite external boundary (boundary consisting of the circle and associated mesh extensions beyond the circle) in most of the designs. W.K. Chorbachi [Cho89]in this context has said that square cannot always be traced out in all designs. In an excerpt from his book: *Tower of Babel: Beyond Symmetry in Islamic Design*, he has written as following:

> "… and finally, there are a few cases of designs where it was absolutely impossible to hide the fact that the analytical method did not hold. These are illustrated (Fig. 4.3) as containing a non-standard zone based as a variation. The elongated rectangular area obviously belongs to a 2-fold symmetry generalisation of the "one way" that is overwhelmingly represented in the square of a 4-fold symmetry group".

In the illustration (see figure 10) W.K. Chorbachi has regarded the rectangle as the external boundary for the design and proved that a square cannot always be possibly marked as the external boundary. However, he too regarded the external boundary as indispensable in a design. But, As has been proved above (see figure 7 and figure 8) the external boundary is Phantom to the Star/Rosette design and its presence cannot be always confirmed until and unless the existence of external mesh can be traced.

6. CONCLUSION

We can conclude that group theories classify arrangements rather than classify unit patterns. This paper presents a viable theorem, which enables us to classify any Star/Rosette based on its gridding attributes. It also generates a classification name for the Star/Rosette, which gives the reader information regarding the Minimum Number of Grids (MNG), and the Lowest Geometric Shape (LSG) used in achieving the design of the IGPs. The results obtained by classifying the Islamic Geometric Pattern can be relative according to our experience. To elaborate, there is always room for a New Critic to classify the same Star/Rosette through a different prism and come out with more normalised gridding than what exists till then, in this case the nomenclature would change to accommodate the findings of the new study.

7. FUTURE SCOPE OF WORK

The Author is currently working on an ambitious computer program developed on Visual Basic platform with inbuilt integration to AutoCAD 2000 through OLE (Object Linking and embedding) technology, that would enable any user of the program to build an IGP from scratch in stages exactly as described in this article and arrive at its classification with ease. The author hopes that the program would be an excellent tool in analyzing IGP's to designers, architects, geometrists and academic fraternity working in related areas.

8. AKNOWLEDGMENT

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