Technology and Geometry in the Design of "Gothic" Vaults in Britain

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Gothic church architecture is characterised by the vaulting systems' dynamic equilibrium in the transverse section, which then guarantees the construction design and safety of bays and their repetition along the main axis of a church. French Gothic is considered as the classic expression of the achievements of the period in Europe but in England, the Early English period (1175-1265) marks the experimentation towards a more national style, where horizontality and verticality are more balanced, resulting in a less ambitious scale of the nave. The technology and role of vaulting systems linked to the advances of the period are reviewed in this work.

The lower height of the transverse section and the less slender lateral walls in Britain mean that the vaulting systems are closer linked to the overall structural behaviour and stability of a church. Moreover, the characteristic linearity of Early English served in design by shafts and ribs can also be interpreted as the masons' wish to highlight the timber origin and the frame-type function of buildings of such scale apparent in contemporary barns. Traditional design methods, such as systems of proportions related to load-bearing element sizes and aspect ratios or the use of precedent buildings as a "manual" or source of construction solutions were possibly developed by the masons in order to codify, disseminate and further develop this empirical knowledge among them.

Understanding the structural behaviour of these buildings makes the assessment of their degree of safety more reliable and this can be further supported by knowledge of the design philosophy of the masons. The efficiency of these traditional methods in vault design and construction is discussed through the structural behaviour of buildings that represent significant advancements in vaulting construction in England, like the cathedrals of Durham, Canterbury, Wells and Lincoln, as well as Westminster Abbey.

The present work focuses on the main vaults in the cathedrals of Durham and Lincoln that can be said to represent the major stags in the evolution of the rib vault in England, as Durham marks the establishment of ribs along the groins and Lincoln indicates the departure from the functional role of the rib in the construction and performance of vaults. The efficiency of their design is examined through structural analysis using Finite Element (FE) analytical models of their main vaults. Ultimately, this analysis will provide a basis where proportions relate with construction types and the role and geometry of the ribs or stone masonry bonds is assessed.

GOTHIC DESIGN

Geometry and design

Stone vaulting became essential in the design of churches of increasing scale throughout the Romanesque and Gothic period due to its efficient use of a relative abundant material like stone (when compared with the use of timber or iron) and its insulating and fire-proofing qualities. The masons knew cross vaults already through Roman basilicas and thermae and tried to copy their spatial and technical qualities: they were invariably generated by combinations of semicircles and built by brick tiles arranged radially. Developments in stone vaulting in Britain and England in particular were driven mainly after the Conquest of 1066, both due to the Normans' advanced technology and their policy for strong religious establishments that could strengthen their position in the country.

Ribs and strengthening techniques were well known to Romans but very rarely expressed on the outer skin of their vaulted structures (as in the case of the Villa dei Sette Bassi outside Rome). The rib however played a major role in the successful experimentation with vaulting systems in the Gothic period and its use was ultimately established in the choir of Durham Cathedral between 1093 and 1133, before the "official" birth of Gothic in St. Denis in Paris in 1130. The experience however showed the spatial and technical limitations of the semicircular arches used then to generate the vault, as will be seen later. The application of the pointed arch, although it existed already, as a combination of two segments of a circle was rapid and allowed wider flexibility in space, improving the geometric and structural compatibility between the intersecting barrels.

The application of ribs and pointed profiles resulted in very systematic design methods that combined geometrical rules and qualitative assessment of the structural behaviour and strength of the building and the vault. Geometric constructions based on the Golden Rule or stable forms like the triangle (the equilateral triangle in particular) allowed the proportions of the intersecting barrels to be fixed in a way that resulted in visual balance, less sharp changes between the shell surfaces and gradually less complicated supports (Fig. 1). These are essential conditions in the stability and high strength of any shell structure.

As the construction technology of vaults matured, ribs would become a sort of permanent part of the formwork upon which the blocks of the cells (the vault proper) would be laid. Both smooth and rebated types of ribs were used and this explains the degree of integrity of the vault when the ribs or the shell fail, as often occurred after shell bombing in the First World War. For a more extensive discussion of construction aspects and the role of the rib see (Theodossopoulos 2004, Fitchen 1961).

No independent structural performance was, however, assigned on the ribs and this is clear on the way they are combined with the vertical shafts carrying on along the columns and piers. The main purpose of this pattern is to highlight continuity in the design and construction between the parts of

a church (Stalley 1999); the presence of shafts can often be the main indicator on whether a vault was planned from the initial set out of the church, as will be seen later in the more detailed discussion of Durham.

Vaults evolved from purely functional priorities to a very wide exploration of the aesthetic possibilities of their main load bearing elements. Variations in the bond of the masonry due to local building conditions led to domical vaults for example, where the blocks are laid normal to the groin (common in Spain) or course rubble that was then plastered (the high vaults of Durham). The unique English typology of fan vaults is a result of emphasis on the role of the springings of vaults, probably a development of the *tas-de-charge* pattern where the ribs are fused together into a single block.

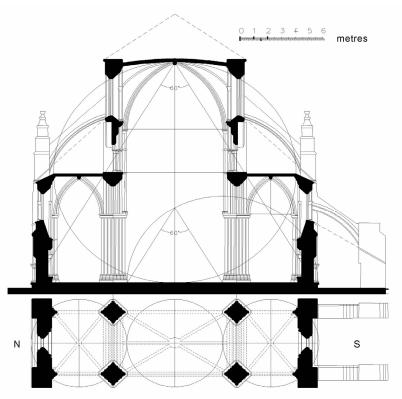


Figure 1. Cross-section of the church of Holyrood Abbey: hypothetical reconstruction showing the use of the equilateral triangle (Theodossopoulos 2004a).

The particular sexpartite arrangement of high vaults in the early Gothic period was an attempt to make the upper structure and enclosure more uniform. The concentration of thrusts however on the main piers was not a good practice and the overall spatial effect was awkward. The main

exploration of the space qualities of vaults occurred in later times mainly through the increase of the intersections or the treatment of ribs as an independent element. The result was a skeletal scheme that ultimately undermined the whole essence of Gothic architecture around structural rigour, ranging from the tierceron vaults to a detached network of ribs as occurred in Germany (*Springgewölbe*).

Structural behaviour of vaults

The most immediate problem the masons faced with the vault in service was the containment of the thrusts, which was usually achieved with flying buttresses abutting the high vaults at spandrel height (Fig. 2 – the upper tier was expected to stabilise wind deformations). Experimental and structural analysis of gothic vaults (Theodossopoulos 2003, 2004a, 2004b) have shown that the dominant action upon a vault is imposed deformations due to geometric instability, usually of the clerestory or aisle walls (Fig. 2). Structural analyses of vaults either within their normal boundary conditions or isolated have demonstrated their high reserves in strength under service loads. Once the abutments are displaced, cracks develop rapidly above the abutments, the groins and the vertex.

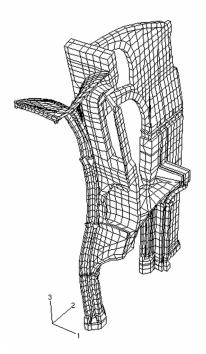


Figure 2. Deformation of a typical bay of a Gothic cathedral (Theodossopoulos 2004b)

The role of the groin is fundamental for the composite action between the intersecting barrel vaults, guaranteeing compatibility between their deformations. Failure of this area can be rapid under the imposed deformations discussed above, resulting in increasingly independent behaviour of each

cell. If movement is assumed to occur in the transverse direction (Fig. 2), the vault in this direction is gradually transformed into a barrel vault supported upon diaphragms and consequently the longitudinal web can be simulated as a series of independent pointed arches, simply supported upon the diagonal ribs.

Tests and analysis of an aisle vault (the S aisle of Holyrood Abbey) indicated failure occurs at 1/30 of the transverse span. Moreover, the analysis showed that a significant part of the weight of the upper structure is carried directly over the nave arch, therefore movement of the abutments causes an earlier failure (at about 1/90 of span). Finally, earth pressures from the spandrel fill in static conditions was proved to be significant only when the extrados of vault is filled to its full height.

GOTHIC VAULTING SYSTEMS IN ENGLAND

As was mentioned earlier, this work focuses on the technology and role of vaulting systems in cathedrals of the Early English period (1175-1265), where innovations from France are adapted into a more national style. The typical characteristic of the churches of this period is more balanced horizontality and verticality and a less ambitious scale of the nave. Emphasis was not placed in the height of the transverse section as in France (the catastrophic collapse of Beauvais in 1264 would have never happened in England), therefore the less slender lateral walls worked closer with the vaulting systems in the structural behaviour and stability of a church. The scale results also in lower lateral thrusts and ultimately a more subtle treatment of the flying buttresses.

Due to historic reasons and the monastic origin of most of the cathedrals, the internal space and layout is fragmented and two transepts are often present. This compartmentation allowed complete reconstructions of the choir or the nave to take place independent from each other, leading to a variety of structural typologies that are almost self-contained and do not disturb each other. Another result of their monastic origin was that often they were built outside the (fortified) city limits, giving more freedom in the layout and the technical solutions of the masons.

Linearity (mainly horizontal) is another characteristic of Early English and is highlighted by shafts and ribs. The masons implicitly employed a vocabulary originating in timber frame-type construction, as developed in contemporary barns, although no transfer of technology is apparent. The masons appear to be conscious of this and often the main load-bearing elements like ribs or the void of the triforium is greatly enhanced. This will become more evident in the subsequent Geometrical Decorated phase, in the treatment of ribs and linear elements (bar tracery etc).

Technical developments

The main technical developments of Early English will be briefly discussed through the structural scheme of the major cathedrals of the period, before focussing on the innovations in Durham and Lincoln Cathedrals.

Durham has a unique place in the history of Gothic in Europe, as the successful application of ribs along the intersections of the barrel vaults was established there, as Bilson demonstrated (Bilson 1922). The design and construction (1093-1133) has followed quite faithfully the project of Bishop William St. Carileph, one of the first Norman bishops of England, to provide a proper shrine for St. Cuthbert. The ground-breaking vaults were first built on the earlier choir, spanning compartments of 8 x 9.9 m (a sexpartite bay could be 15.4 x 9.9 m) but several conflicting decisions followed on whether to build stone vaults in the nave and the transepts as well (Bilson 1922, James 1993).

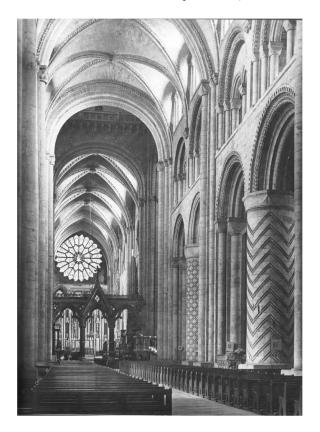


Figure 3. The nave of Durham Cathedral (Pevsner 1985)

The generating arches in both the nave and the aisles are semicircular and they often had to be stilted, creating awkward intersections of the barrel vaults. Moreover, a buttressing system to contain the thrusts of the high vaults was also incorporated within the triforium galleries, in the form of a barrel vault. A careful examination of the construction sequence and changes in the form of the load-bearing elements (Bilson 1922, James 1993) can demonstrate the experimentation of the masons and how this unique application of ribs was incorporated into later phases of the building, as will be summarised later.

Canterbury Cathedral was the next major cathedral that followed the pioneering construction of Durham. The original cathedral was damaged by fire in 1174 and the reconstruction as a shrine of St. Thomas a Becket (+1170) lasted till 1184. This building marks as well the use for the first time of imported spatial and technical solutions, probably by the master mason William of Sens, but the adaptation has not matured yet. This can be seen in the unbalanced proportions of the sexpartite vaults used for the high vaults (Fig. 4) and in areas like the abrupt junction of piers to the upper structure in the choir.

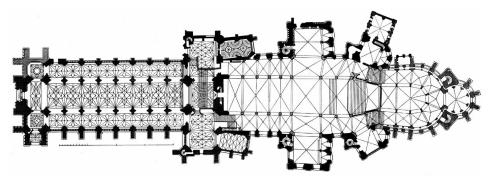


Figure 4. Plan of Canterbury Cathedral (Dehio 1901)

Overall, the aspects of the main load-bearing elements do not demonstrate a full understanding of their functions but a rather decorative treatment. For the first time however linearity is emphasised by leaner ribs and the use of Purbeck shafts, making the link with the vocabulary of timber construction more evident.

Wells Cathedral (together with Lincoln) is considered as the first English Gothic church. The entire design is based on the correct use of the pointed arch and the linear elements are treated with an elegant simplicity that permits focus on their function. The height of the building (20.5 m) and the short construction period (1186-1215) resulted in a uniform project and structural scheme, which probably demonstrated not only the aesthetic but also the technical qualities of the new system in a more efficient manner than its precedents.

Lincoln Cathedral's (1192-1280) ambitious scheme is a free variation of the 1174 scheme for Canterbury. Its quite uniform structural quality, to which all later additions conform, is stressed by the linear pattern and role of ribs in the construction and generation of the vaults, emphasised with Purbeck shafts (Fig. 5). Tierceron vaults (compartment dimensions are $7 \times 13m$), unified by ridge-ribs, were extensively experimented. This established the pattern for similar vaults in England (in the vaults of the Angels' Choir) and showed the potential of liberating shells from symmetric arrangements in the so-called "crazy" vaults in St. Hugh's choir (Kidson 1981).

The trend of returning to contemporary French influences set earlier by Canterbury was carried on at Westminster Abbey, the Norman church restructured in 1245-69 by Henry III. The vaults reach higher (31 m), requiring expressive flying buttresses, while the addition of the chevet at the east end strengthens the building longitudinally. The experiments with tierceron vaults in Lincoln find a full application here in the ribs forming almost a skeleton, marking the transition to (Geometrical) Decorated.

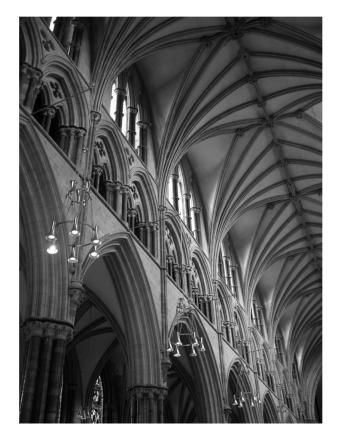


Figure 5. The nave of Lincoln Cathedral

THE INNOVATIONS OF DURHAM AND LINCOLN

These two cathedrals represent major steps in the development and establishment of stone rib vaulting in England. As mentioned earlier, in Durham the rib vaults were successfully applied in vaults of the scale of a nave, while in Lincoln the tierceron vault was developed and a more expressionist approach to shell surfaces was attempted in the "crazy" vaults. Structural analysis of these vaults will be carried out in order to understand the role of their ribs and the degree to which these innovations resulted in a safe enclosure.

	Parallel to bed joint	Normal to bed joint
Material properties	(x-axis)	(y-axis)
Elasticity modulus, E (N/mm ²)	4900	1400
Compressive strength f_c (N/mm ²)	2	6
Flexural strength, F_u (N/mm ²)	0.7	0.2

Table 1. Mechanical properties considered for the stone masonry of the cathedrals

Finite Element (FE) models using shell elements will be generated as they have been earlier developed during the study of the collapse of Holyrood Abbey in Edinburgh (Theodossopoulos 2003, 2004). The models were generated with a CAD pre-processor and then solved with the FE program Abaqus, using shell elements (S4R). The material was initially considered as linear elastic and orthotropic properties were assigned to the webs of the vaults. Due to lack of specific data, it was considered reasonable to use as a first approximation the mechanical properties of the Cathedral of Burgos (Table 1) as they have been assessed in an earlier study (Theodossopoulos 2004b, Hendry 1998).

Cathedral of Christ and Blessed Mary the Virgin, Durham

The major contribution of Durham is the application of stone ribs and examination of the geometry and construction of the vaults shows that they were conceived as an integral part of the structure and were not added once the vault was built. Construction issues of the vault are examined in more detail in the works of Bilson (1922) and James (1993). A fundamental question however is the way the rib was invented and applied on the choir vaults, the first to be built during the remodelling of the cathedral by bishop William. A potential answer was that the masons in Durham explored the construction benefits of the rib as a permanent formwork and mould for the difficult intersection of the barrel vaults along the groins – the structural function of the rib will be realised in later periods, only once the construction methods have matured.

Adherence to the original design (that considered vaulting) is not as apparent as it may seem today since changes in the finances very often prompted substitution of the expensive vaults with timber ceiling, decisions that were later overturned as has been demonstrated in the south transept and probably the nave (Bilson 1922). Moreover, the building provides evidence (James 1993) of the contractors' trials to explore the construction benefits of the rib, such as the successive contradictory decisions to design quadripartite or sexpartite vaults on the bays of the nave. Careful examination of the masonry bonding and the arrangement of the supports, shafts and keystones allows to detect whether the walls were originally designed to receive stone vaulting (as it was not the case with the south transept) as good practice requires vaults to be built together with the walls from which they spring.

The choir was completed in 1104 for the translation of the relics of St. Cuthbert but collapsed probably after 1235 and was replaced. The first phase was concluded with the transepts and the two

first bays of the nave that were required to abut the tower of the crossing (1118). Subsequently, the western aisles and high vaults of the nave were built and the cathedral was completed in 1133. Bilson believes the nave copied the vaults of the choir but backward changes in the design meant provisions for those vaults were not made from the beginning, resulting in an unsatisfactory outcome. He reconstructed the vaults of the choir, based on the evidence of the nave and demonstrated that the original vaults were the first ribbed vaults in Europe.

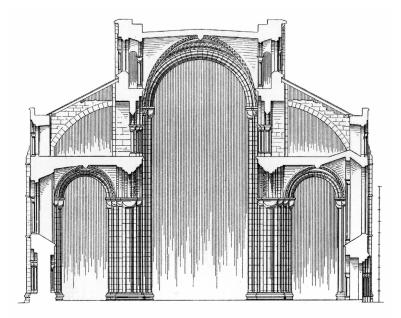


Figure 6. Longitudinal section of the choir, Durham Cathedral (Dehio 1901)

Design was based on the use of semicircles and very often the shell surfaces would intersect in highly twisted planes and lines. This study aims to make a contribution to the understanding of the mason's problems and the solutions they developed by examining the structural behaviour of a typical aisle of the choir and a typical bay of the high vaults of the nave, as they represent the main vaulting types (Fig. 6). Bilson managed to perform a quite accurate survey of the nave during replastering works in 1915 and produced plans and outlines of the geometry of the ribs (Bilson 1922).

A typical plan of the aisle measures 8 x 5.9 m between the axes of the transverse and nave arches, while the height of the vault is 3.4 m. The aisles of the choir are supported on the main pier and the secondary cylindrical pier and since the former is wider the centre of the nave arch is not aligned with the intersection of the diagonals (Fig. 7), resulting in an almost domical vault (Fig. 8). Due to the substantial dimensions of the cross sections of the ribs, they were modelled with shell elements as well. Symmetry was assumed along the transverse edges and the edge along the wall is fully supported.

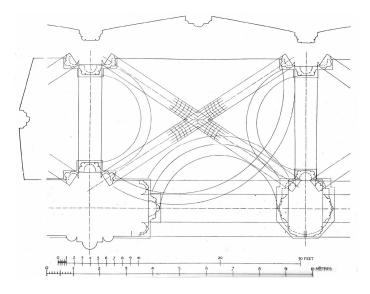


Figure 7. Plan of the aisles of the choir (Bilson 1922)

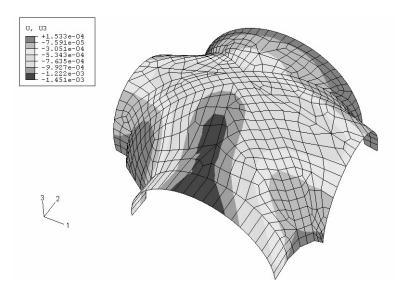


Figure 8. Deflections of the aisle of the choir (in m)

The pattern of the deflections under the self weight of the masonry (considered here as 24 kN/m^3) indicates higher deformations occurring at the front vertex (Fig. 8). The ribs successfully isolate the portion between them and the wall, while the part towards the nave arch is heavily affected also by

the weight of the upper structure. A similar pattern has been observed in other models of aisle vaults (Theodossopoulos 2004a, 2004b), where once again the intersection of the ribs strengthens most of the vault and allows significant deformations only at the front portion. The values in Fig. 8 should be considered to be much lower than in reality as creep, settlement of the piers or unknown interventions might have increased them significantly. The pattern however indicates the role of the ribs and probably the masons might have started to realise the structural benefits as well during construction.

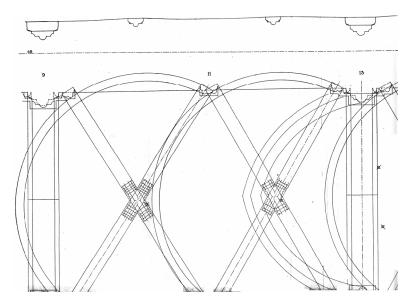


Figure 9. Plan of the high vaults in the nave of Durham Cathedral (Bilson 1922)

A similar analysis was carried out on the high vaults of the nave. The diagonal rib is still a semicircle but it seems the masons have already understood the limitations of this form and designed a pointed arch at the transverse edge, of the same height and radius as the ribs, an essential condition that facilitated the standardisation of the voussoirs that form the arches (Fig. 9). A FE shell model similar to the aisles was generated for these vaults (Fig. 10). Due to the symmetric support along the wall edges the maximum deflections concentrate on the longitudinal vertex, which here is not reinforced by a ridge-rib (Fig. 3).

The longitudinal vertex here is straight and horizontal but the benefits in the visual perception of the nave are not associated with the higher strength of the domical profiles and the deflections are higher between the keys of the ribs, reducing towards the transverse edges due to the deep rib. The examination of the reactions at the supports also indicates an interesting share of the vertical load and thrust between the two extreme supports mainly, with a very small amount carried by the intermediate support. The arrangement of the ribs in the cathedral highlights exactly this minor role

of the support, but it is not clear whether the masons had already developed an intuitive knowledge of the behaviour of the scheme.

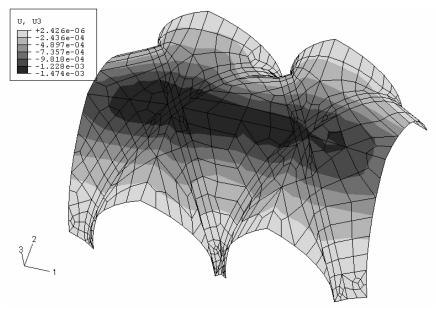


Figure 10. Deflections of the high vaults (in m)

Lincoln Cathedral

The study of Lincoln Cathedral concentrates on the innovative treatment of the ribs and marks the next stage in the development that started with Durham. As was mentioned earlier, ribs start assuming a proper identity in design and are used in abundance either to highlight the junctions (groins, vertices, edges etc) as happens in the early tierceron vaults of the Angels' Choir or to generate completely new surfaces that move away from the symmetric arrangement, liberating thus for the first time the form from function (St. Hugh's Choir). The common factor in both achievements are the lessons from the stability of ribs during construction and the attempt to explore and even highlight the origin of the vaults on this beneficial property.

Like Durham, Lincoln was reconstructed and remodelled in various stages (1192-1280). The first phases started by St. Hugh, after a severe damage caused notoriously by an earthquake on 15 April 1185 – most probably the earthquake precipitated the collapse of an already weak crossing tower over the choir. Reconstruction, beginning with the choir, did not start immediately and this indicates the previous Romanesque nave was not severely damaged. The new choir was planned as a shrine for future pilgrimage use and was ultimately modelled similarly to Becket's Corona in Canterbury (Fig. 4). The octagonal form of the first outline of St. Hugh's choir (the first phase) could be considered as the origin for the geometric explorations throughout the cathedral.

A geometric system of proportions was attempted based on the use of triangles (for example, an equilateral triangle can be inscribed in the elevation of the vault in the later Angels' Choir in Fig. 11). The proportions between nave and aisles are 1:2:1 but the south aisle is narrower and the length of the bay is not constant, which indicates that geometric precision was not a priority (Kidson 1981). In addition, similarities with the precedent of Canterbury are more evident in the structural system adopted: the side structures (aisles and gallery) are treated as a support for the superstructure (Fig. 11) and a 3D model like in Burgos will be required to verify the extent of this scheme (Fig. 2).

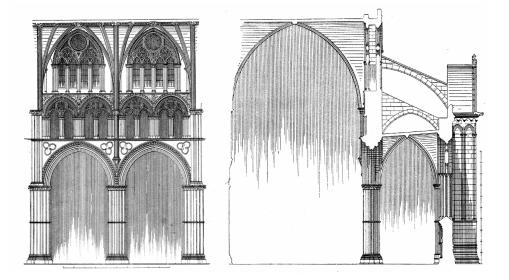


Figure 11. Elevation and cross section of the Angels' Choir, Lincoln Cathedral (Dehio 1901)

The construction quality of the building and the use of Purbeck shafts confirm the uniform aspect of the scheme and construction. That can be said even for the distinct geometries of the so-called "crazy" vaults that have actually initiated the experimental phase that would ultimately lead to the tierceron vaults of the later nave and Angels' choir (Fig. 12). The term was coined by P. Frankl, who wanted to underline the visual and technical conflict between the two keystones that mark the detachment of the two slanted pointed barrel vaults. Despite no direct influence of this form to subsequent designs either in Lincoln or England, this genial experiment embodies the confidence and attempts of the masons to move to forms not strictly related to symmetry or to load-bearing contribution. This expressionist attitude will find more interesting applications in Germany (Kidson 1981) and maybe even indirectly in the new point of view that generated the fan vaults later.

The structural analysis of the "crazy" vaults (Fig. 13) shows slightly higher maximum deflections when compared with the response of a tierceron rib vault over the same compartment (Fig. 14). In both vaults, the ribs have been now modelled with beam elements instead of shells, as they are

leaner and more carefully applied along the intersections. The higher deflections in the Angels' Choir spread on a wider zone along the longitudinal vertex and in both cases there is a strong oneway behaviour of the roof, spanning in the transverse direction between the clerestories.



Figure 12. View of the "crazy" vault in St. Hugh's Choir, Lincoln Cathedral

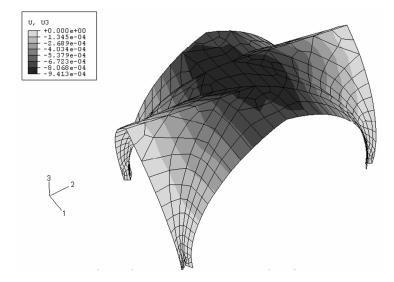


Figure 13. Deflection of the "crazy" vault (in m)

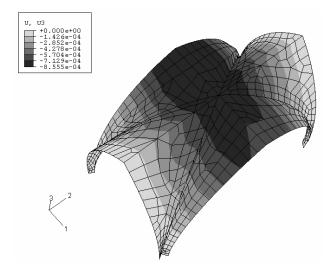


Figure 14. Deflection of the nave vault in Angels' Choir, Lincoln Cathedral

CONCLUSIONS

The first results from a structural analysis of the main vaults of two important buildings in the Early English architecture, Durham and Lincoln, has highlighted some of the structural aspects of the innovations in construction technology brought by each. The revolutionary use of the rib in the Durham Cathedral (1093-1133), whether intentional or the result of trials and errors, enabled the construction of cross vaults at an increasing scale and range of spans while their strengthening effect on the vault gave indications to the masons of further possibilities.

Such possibilities were more maturely explored later in the Lincoln Cathedral (1192-1280) first in the iconoclastic vaults of St. Hugh's Choir. The "liberation" this experiment brought, both in spatial and constructive terms, led to the development of the tierceron rib vaults in the adjacent Angel's Choir, the first of its kind in England. The use of these secondary ribs did not compromise the structural safety of high vaults, which can now be more safely compartmentalised. Next stages in this research project will be to further examine the safety of these structures by applying movement of their supports as it results from lateral instability of the clerestories. Issues of durability of the masonry can also be examined and once the effect of the major actions is better understood through the analysis, aspects of design can be discussed by studying the incidence of variations in the plan and proportions of the vaults.

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